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VERTICAL VIBRATION AND ADDITIONAL DISTRESS OF GROUPED PILES IN LAYERED SOIL

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ABSTRACT

Analytical solutions are developed for calculating the harmonic steady-state axial stiffness, damping, and internal forces of pile groups embedded in multi-layered soil. Pile-soil interaction is represented through a dynamic Winkler model, with frequency-dependent spring and dashpot moduli. Pile-to-pile interaction is taken into account analytically by considering the wave field originating along an oscillating ("source") pile and the diffraction of this field by an adjacent ("receiver") pile. In the case of uniform and two-layer soil profiles, closed-form expressions are derived for both pile impedances and dynamic interaction factors between piles. A solution is finally presented for the additional axial forces developed in piles due to pile-to-pile interaction. The predictions of the model compare well with results of earlier studies, while its simplicity offers a versatile alternative to complicated numerical solutions.

Key words: dynamics, earthquake, impedance, interaction factor, pile, pile group, waves (**IGC**: E8)

INTRODUCTION

Most of the methods developed in the last twenty five years for pile dynamics are of essentially numerical nature (Wolf and Von Arx, 1978; Kaynia, 1982; Waas and Hartmann, 1984; Mamoon et al., 1990). These methods treat the soil as an elastic continuum; they involve discretization of the domain or its boundaries leading to the formation of large systems of equations and therefore to significant computational effort. By contrast, some approximate wave propagation solutions pioneered by Tajimi (1969) and Novak (1974) offer valuable insight into the nature of pile-soil interaction (Takemiya and Yamada, 1981; Nogami, 1983; Konagai and Nogami, 1987; Dobry and Gazetas, 1988). Recent reviews on the subject have been presented by Roesset (1984), Novak (1991), Gazetas et al. (1992), and Pender (1993).

Most of the simple solutions for pile groups make use of the superposition method which is based on interaction factors. This concept, proposed by Poulos in 1968 for static loads, showed that pile group effects can be assessed by superimposing the interaction between only two piles at a time. Static interaction factors would only provide useful information on the dynamic response of a pile group at relatively low frequencies. Indeed, depending on the frequency of vibration, the waves generated along the centerline of an oscillating ("*source*") pile may arrive to the location of an adjacent (hereafter called "*receiver*") pile: (i) with exactly or nearly the same phase, thereby imposing an additional displacement to the receiver pile or, (ii) with phase lag larger than 90° imposing displacements of opposite sign on that pile. Driven by a sequence of successive in- and out-of-phase frequency regions, the dynamic stiffness of pile groups may exhibit large peaks and valleys. This behavior is not observed in solitary piles. The oscillatory nature of dynamic pile group impedances became known in the late 70's, following the study by Wolf and Von Arx (1978).

Kaynia (1982) extended the superposition method to dynamic problems by introducing complex-valued interaction factors accounting for both the amplitude and the phase of the motion transmitted to a pile by a vibrating neighboring pile. Following several subsequent studies, Dobry and Gazetas (1988) derived an approximate closed-form dynamic interaction factor for piles in a homogeneous halfspace. Despite its simplicity, the results of that method were in reasonable agreement with more rigorous solutions (Novak, 1991; Wolf et al., 1992).

On the other hand, pile response in non-homogeneous and layered soil has been the subject of a smaller research effort (Nogami, 1983; Davies et al., 1985; Gazetas et al., 1991; El-Marsafawi et al., 1992; Hijikata and Tomii, 1995). While these studies have shown that soil inhomogeneity may have a profound effect on pile-to-pile interaction, no closed-form expression for interaction factors between piles in layered soil are available in the

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literature.

The distress† of piles in a group has been also scarcely studied. Instead, presently used interaction-factor type of solutions compute the distribution of axial load with depth by considering the piles as solitary, but with the head load determined from the pile group analysis. This, of course, is incorrect because the waves generated by the oscillating piles, apart from inducing additional settlement, would also induce additional stresses, to grouped piles in the form of additional shear tractions along their shafts. Despite the fact that this effect (i.e. the additional distress in piles due to pile-to-pile interaction) has been recognized in some studies dealing with static loads (Poulos, 1968; Randolph and Wroth, 1979), no simple method is presently available to compute it under either static or dynamic conditions.

The main objective of this paper is: (i) to outline a simple method for evaluating the dynamic impedance of single piles in homogeneous soil and to extend it to an arbitrarily multi-layered soil, (ii) to develop a simple physical model for the dynamic interaction factors between piles in multi-layered soil, accounting for the diffraction of the attenuated waves by the "receiver" piles, and (iii) to evaluate the additional distress developed in clustered piles arising from pile-to-pile interaction.

Figure 1 illustrates the problem addressed in this paper: a group of m vertical piles penetrating n soil layers underlain by a deformable base (homogeneous halfspace). The piles are cylinders with length L, diameter d, modulus of elasticity E_p, cross-sectional area A_p, and mass density ρ_p providing mass per unit pile length $m=\rho_p A_p$. They are subjected to a vertical harmonic force $P_G \exp(i\omega t)$ transmitted through a rigid massless pile cap which has no contact with the soil. The soil is modeled as a multi-layered linear viscoelastic material. The typical layer has a thickness h, modulus of elasticity E_s, Poisson's ratio ν_s, mass density ρ_s, and linear hysteretic damping factor β_s introduced through the complex elastic modulus $E_s^*=E_s(1+2i\beta_s)$.

The problem is analyzed in three consecutive steps: (1) response of a single pile, (2) interaction between two piles, (3) response of the pile group.

THE SINGLE PILE

In the seminal work of Novak (1974), the soil surrounding the pile is modelled as a Winkler medium resisting the pile motion by continuously-distributed, frequency-dependent springs, k_z, and dashpots, c_z. The former accounts for the stiffness soil provides to the pile while the latter for the energy losses due to both wave radiation and hysteretic dissipation in the soil. Although approxi-

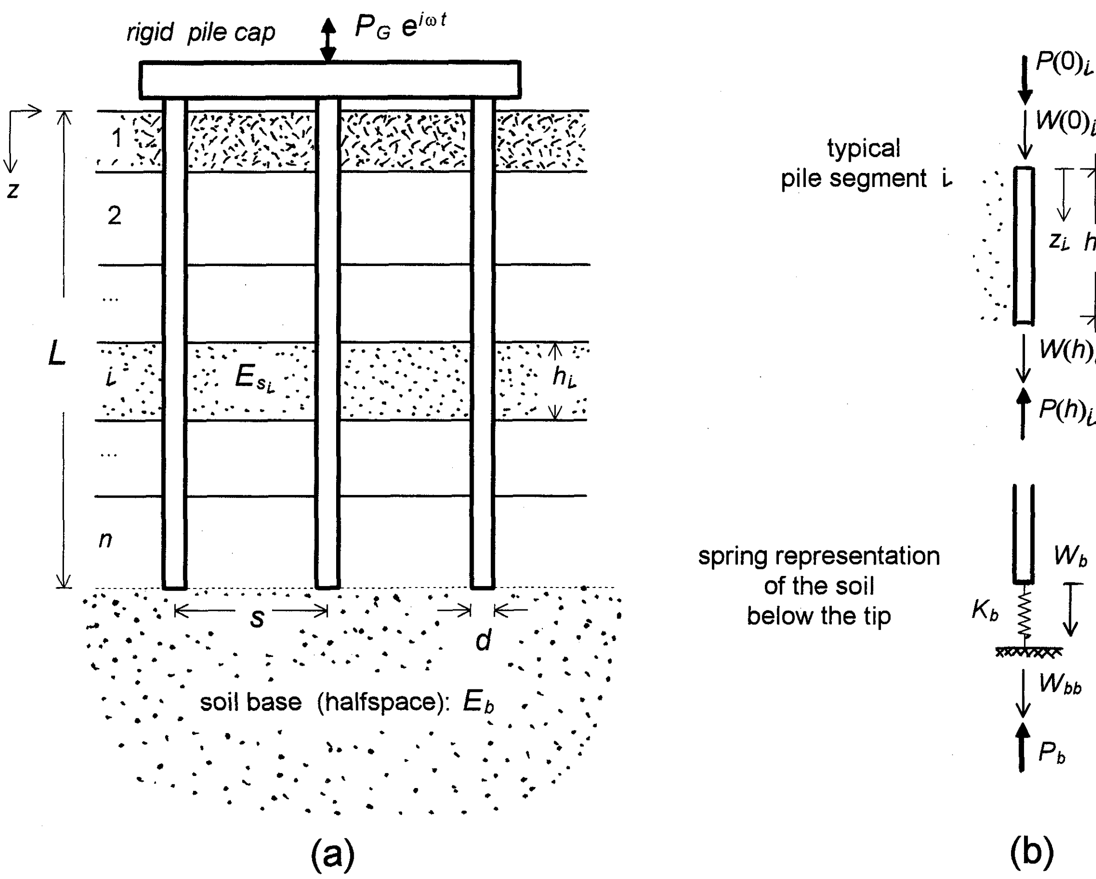

Fig. 1. (a) The problem studied in this Paper. (b) Sign convention

† The word "distress" as used in this paper means the development of axial force and deformation in a pile.

mate, the Winkler model is well accepted because it can easily incorporate: (i) variation of soil properties with depth, and (ii) variation of soil properties with radial distance from the pile due to installation effects and inelastic soil behavior near the pile shaft. This paper studies the effect only of vertical inhomogeneity [factor (i)] on the axial response of piles and pile groups. [Reference is made to Michaelides et al. (1997), for ways to introduce radial inhomogeneity in approximate modeling pile installation and inelastic soil effects on the impedance of single piles and pile groups.]

Based on the Winkler assumption, the complex-valued dynamic impedance at the head of a harmonically-vibrating pile in a homogeneous soil stratum underlain by a deformable base is derived as outlined in Appendix I:

$$\mathscr{R}=E_pA_p\lambda\,\frac{\Omega+\tanh(h\lambda)}{1+\Omega\tanh(h\lambda)} \tag{1}$$

in which the "wave number" λ and the normalized "tip resistance factor" Ω are given by:

$$\lambda=\left[\frac{k_z+i\omega c_z-m\omega^2}{E_pA_p}\right]^{1/2},\quad \Omega=\frac{K_b}{E_pA_p\lambda} \tag{2}$$

K_b is a complex-valued impedance representing the vertical soil reaction at the pile tip. The distributed frequency-dependent spring and dashpots k_z and c_z can be taken from available solutions by Novak et al. (1978), Angelides and Roesset (1980), Gazetas and Makris (1991), and others. This paper utilizes the finite-element-based springs and dashpots of Gazetas and Makris (1991). K_b can be obtained from results published by Veletsos and Verbic (1971), Gazetas (1983), Meek and Wolf (1993), and others. In this work we utilize the solution by Veletsos and Verbic (1971).

In the case of a layered soil, the pile response can be calculated based on the response of a pile segment in a homogeneous layer (*see* Appendix II), by forming $2n$ algebraic equations (n is the total number of soil layers penetrated by the pile) expressing the boundary conditions at the pile top, bottom, and the interfaces. However, an alternative formulation providing closed-form expressions for pile stiffness in multi-layer domains is presented herein. It is based on the repeated use of the "fundamental" single-layer stiffness Eq. (1): starting from the bottom layer n, and proceeding upwards. In a sub-structuring sense, K_n can be considered as a "spring" supporting the tip of the pile segment $(n-1)$. The stiffness atop the segment $(n-1)$ is, therefore, calculated through Eq. (1) but with a modified coefficient Ω equal to $K_n/(E_pA_p\lambda_{n-1})$. This is repeated, layer by layer, until reaching the pile top. The whole procedure is expressed compactly by the following recursive relation:

$$K_i=E_pA_p\lambda_i\frac{\Omega_i+\tanh(h_i\lambda_i)}{1+\Omega_i\tanh(h_i\lambda_i)} \tag{3}$$

in which Ω_i is given by

$$\Omega_i=\frac{K_{i+1}}{E_pA_p\lambda_i} \tag{4}$$

and with boundary conditions

$$K_{n+1}=K_b,\quad \mathscr{R}=K_1 \tag{5}$$

As an example, for a *two-layered* soil stratum, Eqs. (3)–(5) give:

$$\mathscr{R}=E_pA_p\lambda_1\frac{\lambda_2[\Omega+\tanh(h_2\lambda_2)]+\lambda_1[1+\Omega\tanh(h_2\lambda_2)]\tanh(h_1\lambda_1)}{\lambda_1[1+\Omega\tanh(h_2\lambda_2)]+\lambda_2[\Omega+\tanh(h_2\lambda_2)]\tanh(h_1\lambda_1)} \tag{6}$$

Moreover, Eq. (6) can generate the impedance of a pile embedded in a *three-layered* soil by simply substituting:

$$\Omega=\frac{\lambda_3}{\lambda_2}\left[\frac{K_b+E_pA_p\lambda_3\tanh(h_3\lambda_3)}{E_pA_p\lambda_3+K_b\tanh(h_3\lambda_3)}\right] \tag{7}$$

Apparently, explicit expressions such as Eqs. (6) and (7) have some distinct advantages over the numerical solutions. A more complete set of closed-form expressions can be found in the dissertation of Mylonakis (1995).

Pile impedances obtained from Eq. (6) are compared in Fig. 2 with results obtained by the authors using the rigorous formulation of Kaynia (1982). The close agreement between the two methods should not surprise in view of the basic validity of the main assumptions of the Winkler model at high frequencies (Novak, 1991), and the calibration of the spring values with those of rigorous solutions at low ("static") frequencies (Gazetas and Makris, 1991).

DYNAMIC INTERACTION BETWEEN TWO PILES

Following the studies by Poulos (1968), Butterfield and Banerjee (1971), and Randolph and Wroth (1979) on static pile-to-pile interaction, approximate methods were developed (Nogami, 1983; Sheta and Novak, 1982) for the dynamic interaction between piles. These publications were the first attempts to model pile-to-pile interaction based on approximate wave-propagation solutions; they are essentially numerical formulations involving infinite sums of Bessel functions of complex argument and requiring extensive computer calculations.

On the other hand, a simple closed-form dynamic interaction factor has been developed by Dobry and Gazetas (1988). Since that model serves as a starting point for the more elaborate method proposed in this paper, a brief review is worth presenting:

The fundamental idea is that cylindrical waves are emitted from the perimeter of an oscillating "source" pile and propagate in an essentially horizontal direction. This hypothesis is similar to the Winkler assumption of Novak, applied in pile-to-pile interaction rather than to single pile response. The attenuation of vertical soil displacement, $U(s, z)$, at depth z and radial distance s from the pile is written in dimensionless form as (Gazetas and

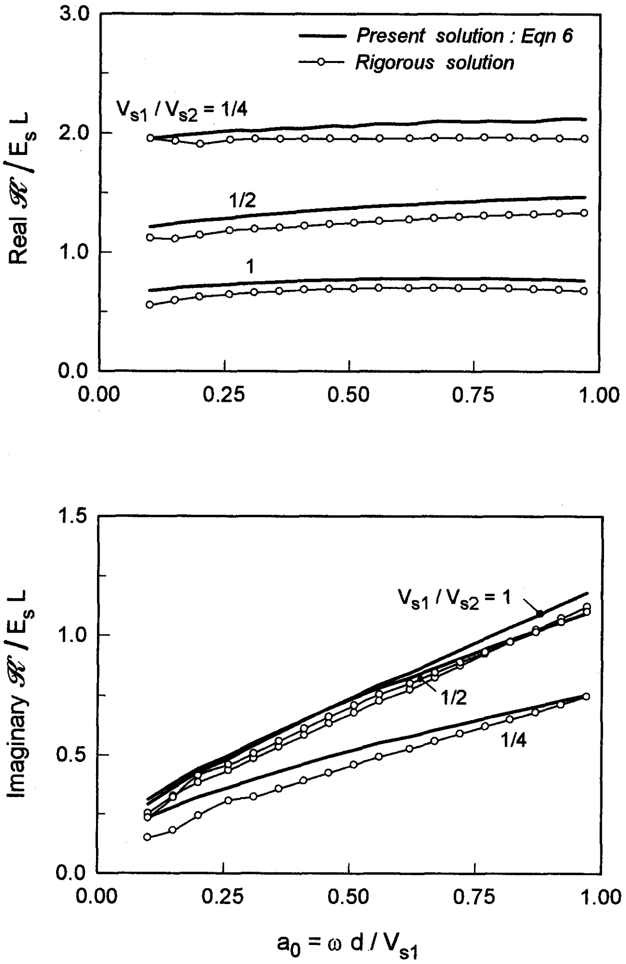

Fig. 2. Dynamic stiffness and damping of a single pile in two-layer soil: comparison of the Winkler solution with results obtained by the authors using the rigorous numerical method of Kaynia (1982): $E_p/E_{s1}=1000$, $L/d=20$, $h_1/L=2/3$, $\nu_s=0.4$, $\rho_p/\rho_{s1}=1.25$, $\rho_{s1}/\rho_{s2}=1.25$, $\beta_{s1}=10\%$, $\beta_{s2}=5\%$

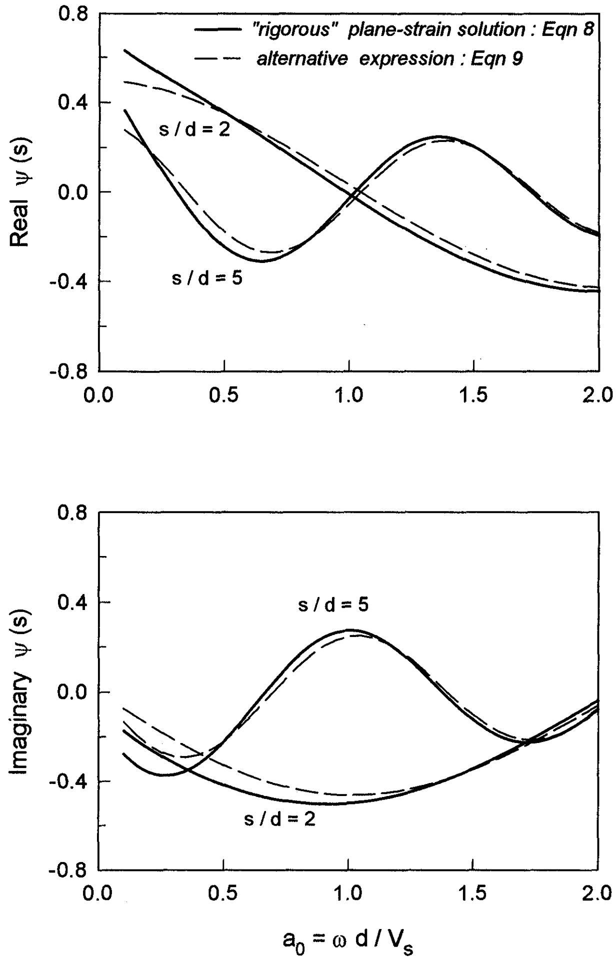

Fig. 3. Attenuation function $\psi(s)$ as function of dimensionless frequency a_0, for two pile separations: comparison between the "rigorous" plane-strain model (Eq. (8)) by Novak and the alternative expression (9), $\beta_s=5\%$

Makris, 1991):

$$\psi(s) \equiv \frac{U(s,z)}{U\left(\dfrac{d}{2}, z\right)} = \frac{H_0^{(2)}\left(\dfrac{s}{d}\dfrac{a_0}{\sqrt{1+2\,i\beta_s}}\right)}{H_0^{(2)}\left(\dfrac{1}{2}\dfrac{a_0}{\sqrt{1+2\,i\beta_s}}\right)} \tag{8}$$

in which $a_0=\omega d/V_s$ and $H_0^{(2)}(\;\;)$ is the Hankel function of zero order and second kind and β_s is the hysteretic damping ratio of the soil. A simpler alternative to Eq. (8) is

$$\psi(s) \approx \left(\frac{2s}{d}\right)^{-1/2} \exp\left[-(\beta_s+i)\left(\frac{s}{d}-\frac{1}{2}\right)a_0\right] \qquad (9)^{\dagger}$$

The real and imaginary parts of Eqs. (8) and (9) are contrasted in Fig. 3 for two different pile spacings. Their agreement is very good and, as it will be shown later in this paper, use of either function gives satisfactory predictions for the interaction factors.

In order to generate an interaction factor, Dobry and Gazetas (1988) introduced the following assumptions:

1. The pile is stiff enough compared with the surrounding soil (say, $E_p/E_s>500$), so that the gradient of pile displacement with depth is relatively small. This ensures the (approximate) validity of the plane-strain Eqs. (8)–(9).
2. Pile rigidity ensures that the cylindrical waves emanate simultaneously from all points along the pile length, even if not with identical amplitudes. Therefore, in the case of a vertically homogeneous soil, the outward spreading waves "strike" an adjacent pile simultaneously along its shaft.
3. A "receiver" pile, located at a distance s from the oscillating "source" pile, follows exactly the attenuated soil motion given by Eqs. (8)–(9). This implies that no interaction was considered between the receiver pile and the surrounding soil.

With these assumptions, the dynamic interaction factor is simply equal to the attenuation function $\psi(s)$:

† Equation (9) is slightly different than that originally proposed by Dobry and Gazetas (1988). The constant $(-1/2)$ appearing in the exponential term of Eq. (9) has been introduced to better match the boundary condition $\psi(s=d/2)=1$.

$$\alpha \approx \psi(s) \tag{10}$$

Although just an approximation, this simple wave model gives very satisfactory results for relatively stiff piles in homogeneous soil (Dobry and Gazetas, 1988; Novak, 1991; Wolf et al., 1992). In fact, the effectiveness of the model is rather surprising, especially if one recalls that several important parameters (i.e., the pile slenderness ratio, L/d, and the pile-soil stiffness ratio, E_p/E_s) are not included in the model. However, the validity of Eq. (10) gradually deteriorates with both increasing inhomogeneity of the soil and increasing slenderness of the pile.

A New Proposed Model for Pile-to-Pile Interaction

To overcome the limitations of the Dobry-Gazetas (1988) model—in particular those arising from assumptions (2) and (3)—an improved model is presented herein, with reference to Fig. 4. It involves the following three steps:

STEP 1: The complex-valued displacement profile along a single "source" pile, $W_{11}(z)$, in a layered soil is determined using the procedure described in the preceeding section or any other analytical approach (a transfer-matrix formulation is outlined in Appendix II).

STEP 2: Cylindrical waves are generated from each depth along the "source" pile with initial amplitude and phase equal to the complex function $W_{11}(z)$. With the soil consisting of a number of homogeneous horizontal layers, it is assumed that the waves propagate in an essentially horizontal manner within each layer. This implies that the radial spreading of waves, although different for each layer, still obeys the cylindrical attenuation law of Eqs. (8)-(9). Under these assumptions, the "free-field" vertical soil displacement at a depth z and distance s from the periphery of the source pile is given by:

$$U(s, z)_j \approx \psi(s)_j W_{11}(z)_j \tag{11}$$

in which j denotes the number of soil layer. (This is in essence a "separation of variables" approximation.)

STEP 3: If a vertical "receiver" pile carrying no load at its head is located at a radial distance s from the source pile, it does not follow the free-field motion of Step 2; its axial stiffness combined with the soil reaction at the pile base *gives rise to an interaction between this ("receiver") pile and the surrounding soil, leading to diffraction of the arriving wave field*. Thereby, pile displacement is different (usually smaller) than the free-field displacement given by Eq. (11). To account in a simple yet realistic way for this diffraction, the receiver pile is modelled as a Winkler beam in which the excitation $U(s, z)$ is applied at the supports of the distributed soil springs and dashpots. The mechanics of this loading is in a sense the reverse of that of Step 1. In Step 1 the "source" pile in-

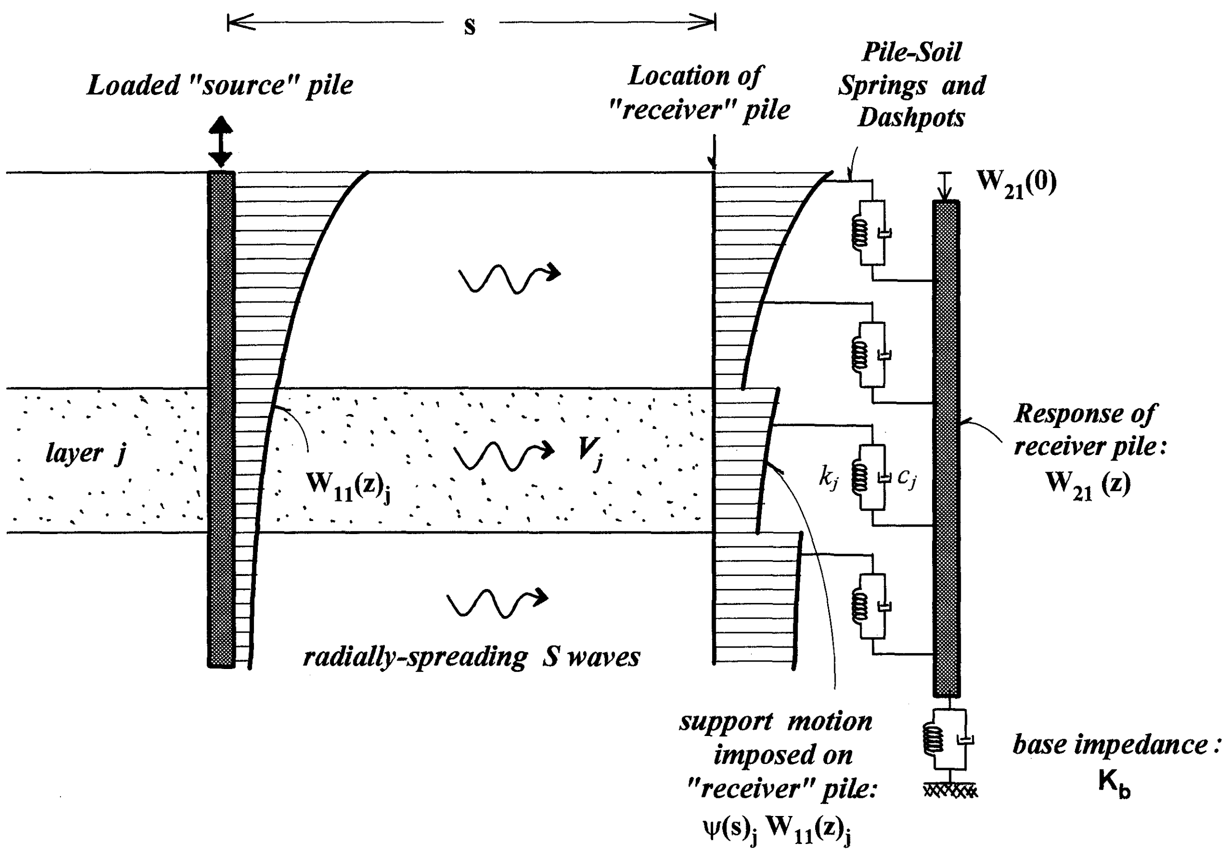

Fig. 4. Schematic illustration of the proposed model for computing the influence of a head-loaded "source" pile upon the adjacent "receiver" pile carrying no load at its head, in layered soil. The response of the head "receiver" pile to a unit displacement of the head of the source pile at a specific frequency defines the interaction factor between the two piles

duces displacements in the soil through its "reacting" springs and dashpots, whereas in Step 3 the soil induces displacements on the receiver pile through its "transmitting" Winkler springs-dashpots. A similar 3-step model was proposed by Makris and Gazetas (1992) who studied the lateral interaction of infinitely-long fixed-head piles in homogeneous soil. In this work we analyze the axial interplay between two piles, accounting for finite pile length and layered soil.

For a receiver pile the dynamic equilibrium of an infinitesimal pile segment yields the governing equation:

$$E_pA_p\frac{d^2W_{21}(z)}{dz^2}+m\omega^2W_{21}(z)-(k_z+\mathrm{i}\omega c_z)\times[W_{21}(z)-U(s,z)]=0 \tag{12a}$$

where

$$U(s,z)=\psi(s)W_{11}(z)=\psi(s)(A_{11}e^{\lambda z}+B_{11}e^{-\lambda z}) \tag{12b}$$

is the motion induced by the source pile, obtained for each soil layer from Step 2; A_{11} and B_{11} are integration constants to be determined from the boundary conditions of the source pile. The solution of Eqs. (12a) is:

$$W_{21}(z)=\frac{k_z+\mathrm{i}\omega c_z}{2\lambda E_pA_p}\psi(s)z(-A_{11}e^{\lambda z}+B_{11}\mathrm{e}^{-\lambda z})+A_{21}e^{\lambda z}+B_{21}e^{-\lambda z} \tag{13}$$

in which A_{21} and B_{21} are new integration constants to be calculated from the boundary conditions of the receiver pile (i.e., zero force at the pile head and continuity of force and displacement at the various interfaces); E_p and A_p are the modulus of elasticity and the cross sectional area of the pile, respectively.

As a first application, the response of a receiver floating pile in a homogeneous profile is obtained in closed form:

$$W_{21}(z)=\frac{k_z+\mathrm{i}\omega c_z}{k_z-m\omega^2+\mathrm{i}\omega c_z}\frac{\psi(s)}{2}\left[\frac{K}{E_pA_p\lambda}(\lambda z\cosh(\lambda z)-\sinh(\lambda z))-\lambda z\sinh(\lambda z)+\frac{2h\lambda+\sinh(2h\lambda)+\Omega^2[\sinh(2h\lambda)-2h\lambda]+2\Omega[\cosh(2h\lambda)-1]}{\sinh(2h\lambda)+\Omega^2\sinh(2h\lambda)+2\Omega\cosh(2h\lambda)}\cosh(\lambda z)\right]W_{11}(0) \tag{14}$$

The ratio $W_{21}(z=0)/W_{11}(z=0)$ is by definition the desired interaction factor; it can be written as a product of two complex-valued functions:

$$\alpha=\psi(s)\zeta(h\lambda,\Omega) \tag{15a}$$

where the function $\zeta=\zeta(h\lambda,\Omega)$ is given by

$$\zeta=\frac{k_z+\mathrm{i}\omega c_z}{k_z-m\omega^2+\mathrm{i}\omega c_z}\frac{2h\lambda+\sinh(2h\lambda)+\Omega^2[\sinh(2h\lambda)-2h\lambda]+2\Omega[\cosh(2h\lambda)-1]}{2\sinh(2h\lambda)+2\Omega^2\sinh(2h\lambda)+4\Omega\cosh(2h\lambda)} \tag{15b}$$

Of these two functions, ψ denotes the induced *free-field* displacement, while ζ *stands for the effect of the diffraction of the arriving wave field due to the rigidity of the pile and the interaction between pile and surrounding soil* [Note that Dobry and Gazetas (1988) neglected pile-soil interaction (i.e. Step 3) and (implicitly) assumed $\zeta=1$.]

For an end-bearing pile, $\Omega\rightarrow\infty$, Eq. (15b) simplifies to:

$$\zeta=\frac{k_z+\mathrm{i}\omega c_z}{k_z-m\omega^2+\mathrm{i}\omega c_z}\frac{\psi(s)}{2}\left[1-\frac{2h\lambda}{\sinh(2h\lambda)}\right] \tag{16}$$

For the special case that the pile is completely free of reaction stress at its tip (i.e., not supported by soil), Ω vanishes and

$$\zeta=\frac{k_z+\mathrm{i}\omega c_z}{k_z-m\omega^2+\mathrm{i}\omega c_z}\frac{\psi(s)}{2}\left[1+\frac{2h\lambda}{\sinh(2h\lambda)}\right] \tag{17}$$

For an infinitely-long pile, $h\rightarrow\infty$, all the preceding equations converge to the remarkably simple expression:

$$\zeta=\frac{k_z+\mathrm{i}\omega c_z}{k_z-m\omega^2+\mathrm{i}\omega c_z}\frac{\psi(s)}{2} \tag{18}$$

The static values of Eq. (15b) are plotted in Fig. 5 for various "tip resistance factors", Ω, in terms of the dimensionless function $\zeta=\alpha/\psi(s)$. Note that ζ is always less than unity expressing solely pile-to-soil interaction, and approaches 1/2 as pile length increases towards infinity. The basic expression of the simplified Dobry-Gazetas (1988) model, $\zeta=1$, is also depicted on the graph. Notice the symmetry of the interaction factor of a pile unsupported at the tip ($P_b=0$) to that of a fully end-bearing ($W_b=0$) pile, with respect to the infinitely-long pile. [The average of the two factors equals the third, as is evident in Eqs. (16)–(18).] Further discussion on static pile-to-pile interaction is given in Mylonakis and Gazetas (1997).

Interaction between Pile Bases

In addition to the shaft-to-shaft pile interaction, a displacement field exists around the pile base. As a result, interaction will also develop between pile bases. Its importance is examined herein. Assuming that the pile base behaves as a rigid circular disk on the "surface" of the underlying halfspace, the soil displacement attenuates away from the pile tip in the following way (Meek and Wolf, 1993):

$$\psi_b(s)\equiv\frac{U(s,L)}{W_b}\approx\frac{d}{\pi s}\exp\left[-(i+\beta_s)\frac{V_s}{V_R}a_0\left(\frac{s}{d}-\frac{1}{\pi}\right)\right] \tag{19}$$

in which V_R denotes the propagation velocity of Rayleigh waves.

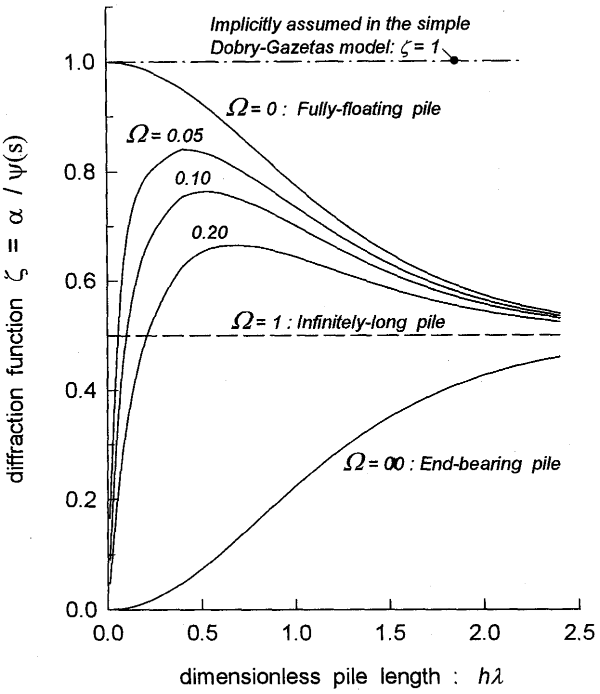

Fig. 5. Normalized pile-to-pile interaction factor in terms of the developed "diffraction" function $\zeta=\alpha/\psi(s)$ [Eq. (15b)], for piles embedded in single-layer soil and for different support conditions at the pile tip

Assuming that the attenuated settlement $\psi_b(s)W_b$ acts on the base of the soil spring supporting the tip of the receiver pile (Fig. 4) the new boundary condition at the pile tip would give rise to a complementary base-to-base interaction factor, $\alpha_b(s)$. As an example, in the particular case of homogeneous soil, $\alpha_b(s)$ can again be expressed as the product of two functions:

$$\alpha_b=\psi_b(s)\zeta_b(h\lambda,\ \Omega) \tag{20a}$$

where the function $\zeta_b=\zeta_b(h\lambda,\ \Omega)$ is given by (Mylonakis, 1995)

$$\zeta_b=\frac{2\Omega}{2\Omega\cosh(2h\lambda)+\sinh(2h\lambda)(\Omega^2+1)} \tag{20b}$$

The overall interaction factor between the piles can be taken approximately equal to the sum of the shaft-to-shaft component α plus the base-to-base component α_b (an idea introduced by Randolph and Wroth, 1979, for pile statics.)

It is easy to show that, for the frequency range of interest: (i) the magnitude of $\psi_b(s)$ is controlled by the static term, $d/(\pi s)$, which decreases linearly with radial distance s, and (ii) ζ_b is much smaller than unity. Thus, for $L/d\approx 20$ and $s/d\approx 2$, Eq. (20a) gives α_b magnitudes of the order of 10^{-2} or less, while for larger spacings α_b is even smaller. This means that base-to-base interaction be-

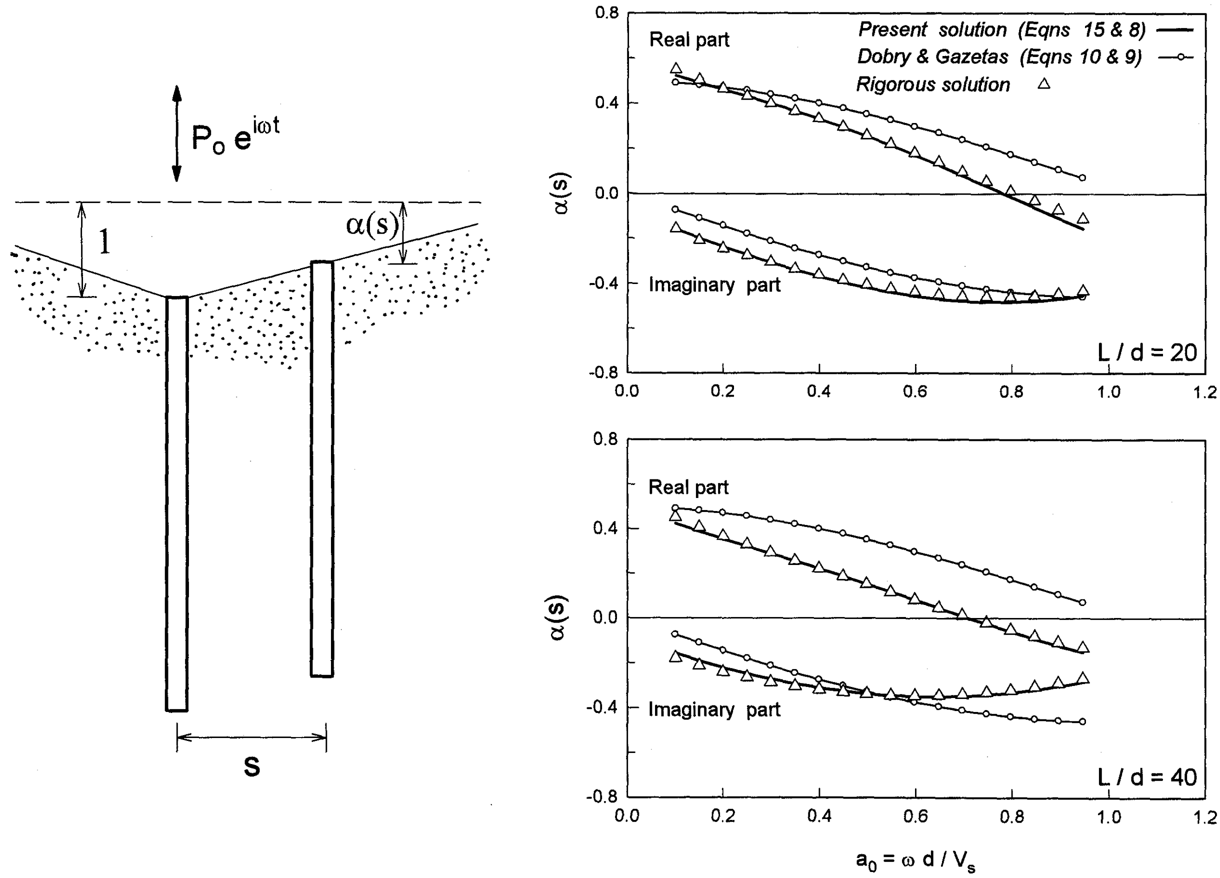

Fig. 6. Interaction factors between two piles in homogenous soil with $s/d=2$ and $L/d=20$, 40: comparison of the results obtained by (i) the developed Eq. (15), (ii) the simplified solution of Dobry and Gazetas [Eq. (10)], and (iii) results obtained by the authors using the rigorous method of Kaynia (1982), $E_p/E_s=1000$, $\nu_s=0.4$, $\rho_p/\rho_s=1.25$, $\beta_s=5\%$

tween two adjacent piles is obviously *negligible* and will not be further discussed in this paper.

Figure 6 compares the interaction factors as functions of frequency, for a pair of closely-spaced piles ($s/d=2$) in homogeneous soil, obtained: (i) with the proposed method [Eq. (15)], (ii) with the Dobry-Gazetas (1988) model [Eq. (10)], and (iii) with the rigorous formulation of Kaynia (1982). The Dobry-Gazetas interaction factor predict quite well the interaction between the short ($L/d=20$) piles, but its performance deteriorates for longer ($L/d=40$) piles. By contrast, the method developed herein, taking into account the interaction between soil and passive pile (i.e. Step 3), leads to an excellent comparison with the rigorous results.

The accuracy of the proposed solution is further illustrated in Figs. 7 and 8 for three different pile-to-pile spacings and the same soil conditions as in Fig. 6. Equation (15), associated with either Eq. (8) or Eq. (9), matches reasonably well the rigorous results in the whole frequency range $0<a_0<1$.

Pile-to-Pile Interaction in Multi-Layered Soil

In the spirit of the recursive expression developed for the single-pile impedance, an analogous formulation is presented herein providing closed-form interaction factors in multi-layered soil. To this end, the arbitrary segment *i* of the receiver pile is considered free head, supported at the base by a "spring" representing the stiffness of the receiver pile below this segment. This element is excited:

(1) along its shaft by the corresponding element *i* of the source pile
(2) at its base by the displacement transmitted from below (i.e., due to the response atop the "receiving" element $i+1$).

Combining these two effects leads to a "segmental" interaction factor α_i (i.e. atop segment i), recursively expressed as a function of α_{i+1}:

$$\alpha_i=\zeta(h_i\lambda_i,\ \Omega_i)\psi(s)_i W_{11}(0)_i + \alpha_{i+1}[\cosh(h_i\lambda_i)-\sinh(h_i\lambda_i)\Omega_i] \tag{21}$$

where ζ is the diffraction function (Eq. 15b). $W_{11}(0)_i$ is the pile displacement atop the "*source*" segment *i*, determined in Step 1. The first term in Eq. (21) expresses component (1) while the second term expresses component (2). Ultimately, however, both components express the in-

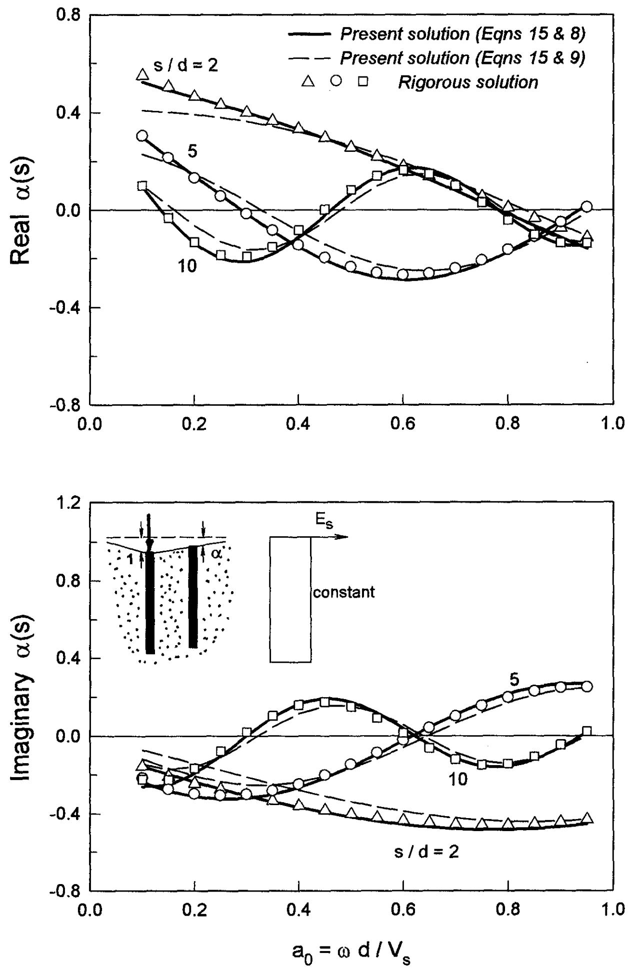

Fig. 7. Interaction factors between two piles in homogeneous soil for various pile separations s/d: comparison of the derived Eq. (15) with results obtained using the rigorous solution of Kaynia (1982), $L/d=20$, $E_p/E_s=1000$, $\nu_s=0.40$, $\rho_p/\rho_s=1.25$, $\beta_s=5\%$

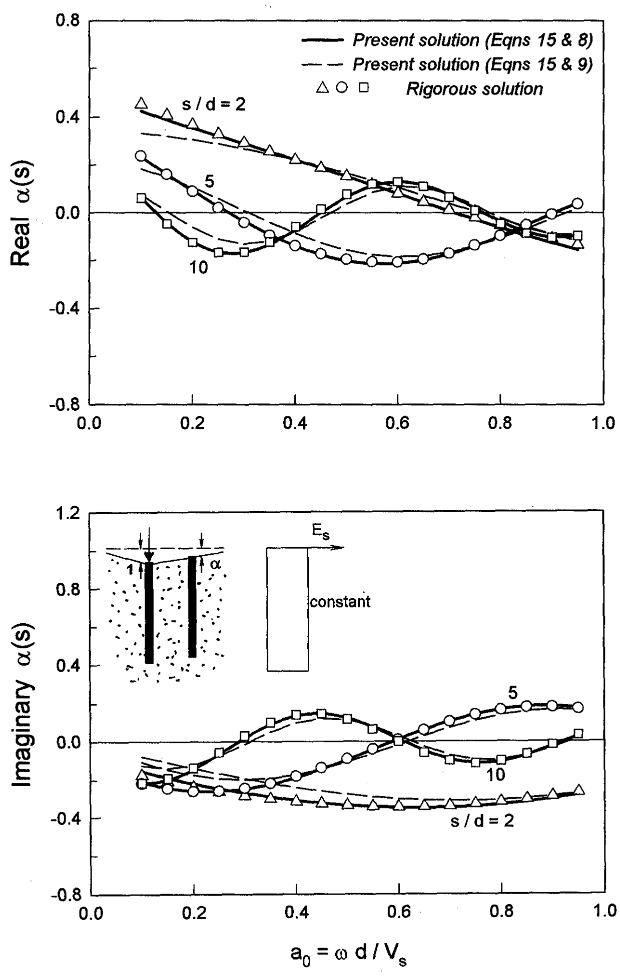

Fig. 8. Interaction factors between two piles in homogeneous soil for various pile separations s/d: comparison of the derived Eq. (15) with results obtained using the rigorous solution of Kaynia (1982), $L/d=40$, $E_p/E_s=1000$, $\nu_s=0.40$, $\rho_p/\rho_s=1.25$, $\beta_s=5\%$

teraction between the *shafts* of the piles. Starting from pile bottom, Eq. (21) is applied repeatedly, layer after layer, until reaching the pile top, whence the desired interaction factor is obtained.

As an example, for two interacting piles in a *two-layered soil*, Eq. (21) gives the following analytical expression for the interaction factor:

$$\alpha=\psi_1(s)\zeta(h_1\lambda_1, \Omega_1)+\psi_2(s)\zeta(h_2\lambda_2, \Omega)\times\frac{2\Omega_1}{2\Omega_1\mathrm{Ch}\,(2h_1\lambda_1)+\mathrm{Sh}\,(2h_1\lambda_1)(\Omega_1^2+1)} \qquad \text{(22a)}$$

where

$$\Omega_1=\left[\frac{\lambda_2}{\lambda_1}\frac{\Omega+\mathrm{Th}\,(h_2\lambda_2)}{1+\Omega\,\mathrm{Th}\,(h_2\lambda_2)}\right] \quad \text{and} \quad \Omega=\frac{K_b}{E_pA_p\lambda_2} \qquad \text{(22b)}$$

Sh ( ), Ch ( ), and Th ( ) stand for the hyperbolic trigonometric functions sinh ( ), cosh ( ), and tanh ( ) respectively. It is noted that, in the realm of the simplifying assumptions of the model, Eq. (22) is an "*exact*" solution. By setting $h_1=0$, Eq. (22) duly reduces to the single-layer Eq. (15).

In Figs. 9 to 11, Eq. (22) is graphically compared against results obtained by the authors using the rigorous numerical formulation of Kaynia (1982). Figure 9 refers to a *two-layered* soil profile with $V_{s1}/V_{s2}=1/2$ (which corresponds to a ratio of shear moduli of about 4), and a pile "embedment" ratio $h_1/L=2/3$. Compared with the homogeneous soil (Fig. 7), the presence of the stiff bottom layer makes the interaction factor considerably smaller and less sensitive to frequency. Evidently, the proposed method (using either Eq. (8) or Eq. (9)) reproduces adequately the details of the rigorous curves.

In Fig. 10, V_{s1}/V_{s2} equals 1/4, i.e. the lower layer is somewhat more than 16 times stiffer than the upper. The interaction curves are quite flat, equal to merely 30%–40% of the corresponding "homogeneous" values (*see* Fig. 7). Eq. (22) is again in satisfactory accord with the numerical results.

Figure 11 refers to a continuously inhomogeneous profile is examined with modulus proportional to depth; at the tip ($z=20d$): $E_s(L)=E_p/500$. To analyze pile-to-pile interaction, the analytical expression Eq. (21) is utilized with the soil profile discretized in 10 homogeneous

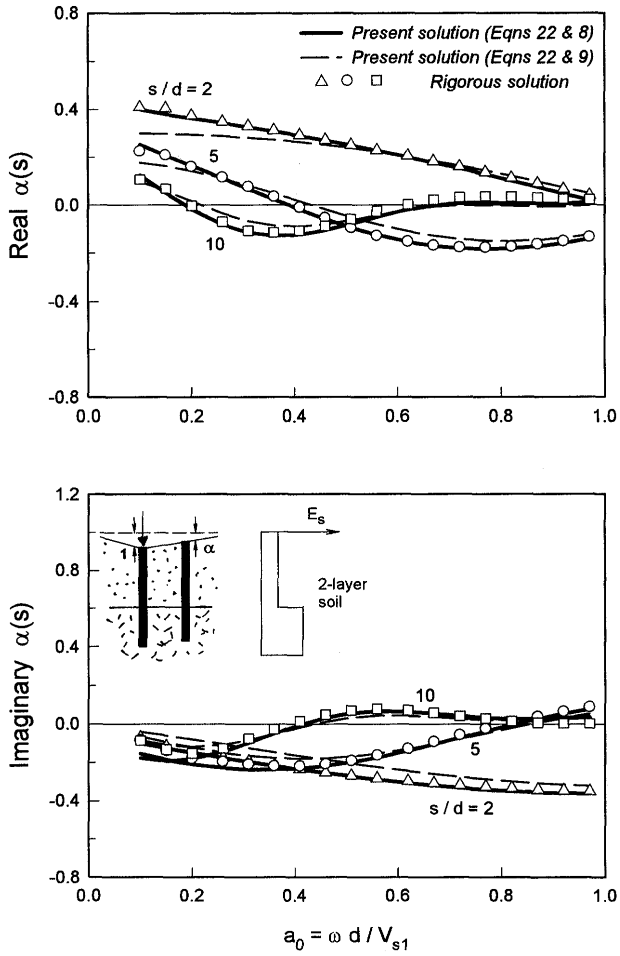

Fig. 9. **Interaction factors for piles in two-layer soil, $V_{s1}/V_{s2}=1/2$, for various pile separations s/d: comparison of the derived Eq. (22) with results obtained using the rigorous solution of Kaynia (1982), $L/d=20$, $h_1/L=2/3$, $E_p/E_{s1}=1000$, $\nu_s=0.40$, $\rho_p/\rho_{s2}=1.25$, $\rho_{s1}/\rho_{s2}=0.80$, $\beta_{s1}=10\%$, $\beta_{s2}=5\%$**

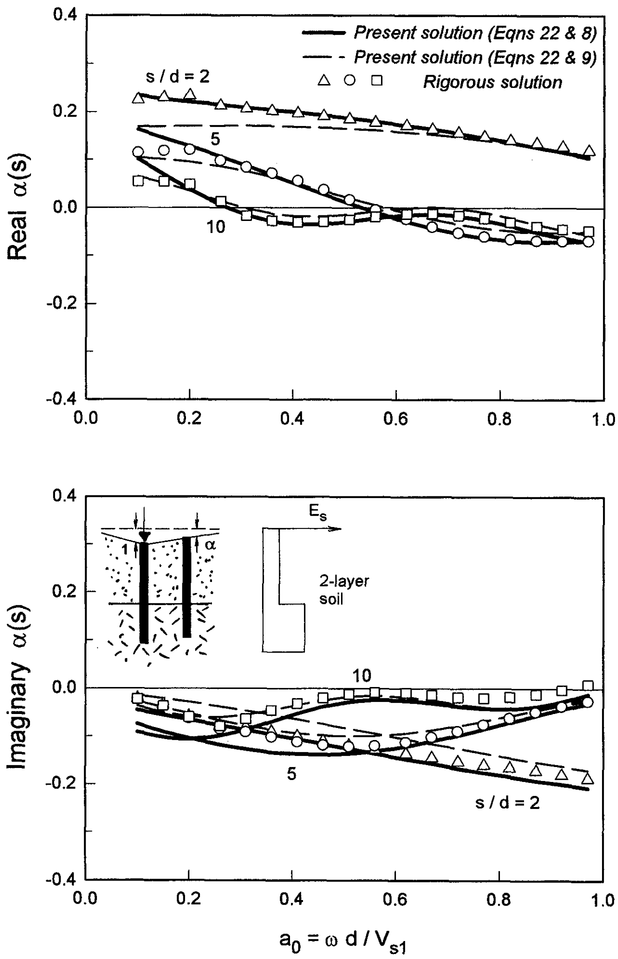

Fig. 10. **Interaction factors for piles in two-layer soil, $V_{s1}/V_{s2}=1/4$, for various pile separations s/d: comparison of the derived Eq. (22) with results obtained using the rigorous solution of Kaynia (1982), $L/d=20$, $h_1/L=2/3$, $E_p/E_{s1}=1000$, $\nu_s=0.40$, $\rho_p/\rho_{s2}=1.25$, $\rho_{s1}/\rho_{s2}=0.80$, $\beta_{s1}=10\%$, $\beta_{s2}=5\%$**

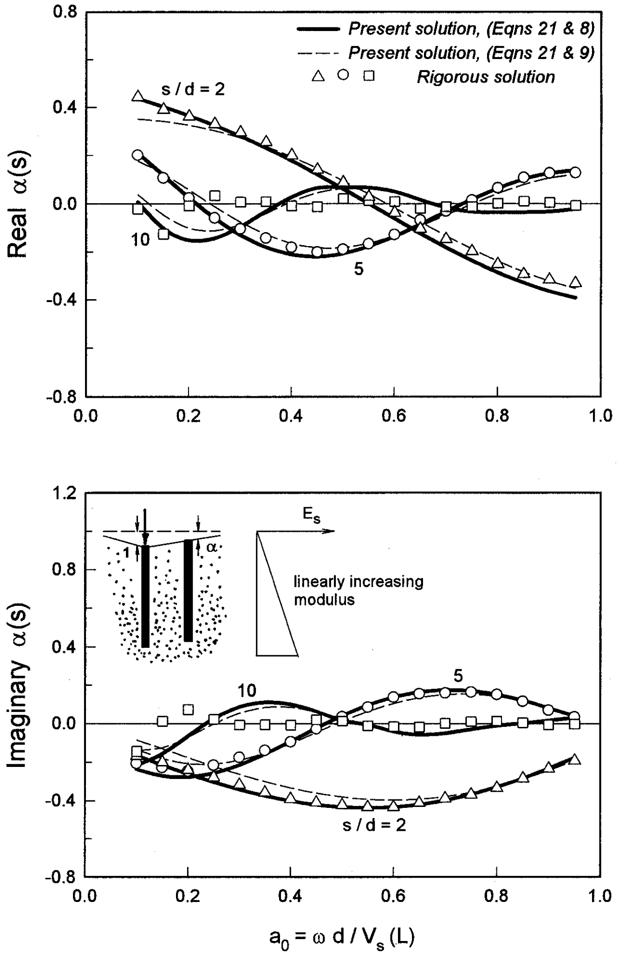

Fig. 11. Interaction factors between two piles in a inhomogeneous soil with linearly increasing modulus $E_s(z)=E_p z/500L$: comparison of the present solution versus results obtained using the rigorous method of Kaynia (1982), $L/d=20$, $\nu_s=0.40$, $\rho_p/\rho_s=1.50$, $\beta_s=5\%$

layers. Despite that the simplifying assumption of purely-horizontal wave propagation may in principle be less accurate in this case, for $s/d=2$ and $s/d=5$ the comparison with the rigorous results is very good. This means that at such distances the effect of the likely curvature of the wave rays is not significant. In contrast, for $s/d=10$ and $a_0>0.20$, the influence of the non-horizontal wave propagation becomes evident: while the rigorous interaction factors are essentially zero, the proposed solution predicts values between $+0.10$ and -0.10. Fortunately, at such large distances, pile-to-pile interaction is too small for the differences to be of major significance.

RESPONSE OF PILE GROUPS

According to the superposition method, the response of m identical piles connected through a rigid cap is written as (Poulos, 1968):

$$P_G=\sum_{i=1}^{m} P_i=[\{1\}^T[A]^{-1}\{1\}]D_G=\mathscr{R}_G D_G \tag{23}$$

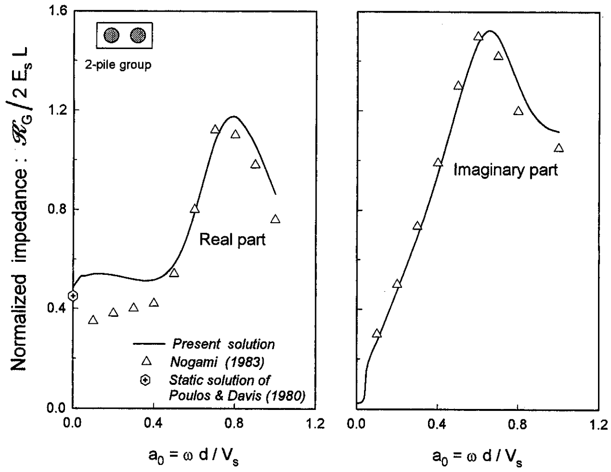

Fig. 12. Vertical dynamic stiffness and damping of 2 rigid piles spaced at $s=5d$ in homogeneous soil on rigid rock at $H=2L$: comparison with: (i) the static solution of Poulos, and (ii) the approximate dynamic plane-strain solution of Nogami; $L/d=37.5$, $\nu_s=0.40$, $\rho_p/\rho_s=1.50$, $\beta_s=2.5\%$

in which: P_G is the load of the cap; A is an m by m complex matrix with $A(i, j)$ being equal to the interaction factor between the piles i and j, divided by the stiffness of the single pile; $D_G=D_i$ is the cap displacement and $\mathscr{R}_G$ is the complex impedance of the group.

Figure 12 compares the proposed method with results of the plane-strain solution of Nogami (1983) for two perfectly-rigid piles embedded in a homogeneous stratum of thickness $H=2L$ and spaced at $s=5d$. Since both methods are based on superposition of cylindrical wave fields, their predictions at the high-frequency range ($a_0>0.5$) are very similar. In the low-frequency range, however, the predictions for the real part of the impedance diverge. The solution of Nogami underestimates the group stiffness leading eventually to zero static stiffness. By contrast, the proposed method seems to provide a satisfactory static stiffness, as inferred from the agreement with the static solution of Poulos and Davis (1980).

Figure 13 refers to the group efficiency factor [defined as the dynamic impedance of the pile group divided by the sum of the individual static stiffnesses of the piles]. The figure compares the results obtained with the proposed method against the boundary-element results of Mamoon et al. (1990), for a 3×3 pile group in homogeneous halfspace. The two methods are in good accord, although for $s/d=5$ the proposed method gives slightly smoother peaks.

The significance of soil layering on the impedance of a closely-spaced 3×3 pile group is illustrated graphically in Fig. 14 for a two-layered soil. In addition this figure contrasts the results calculated with the interaction factors from in Figs. 7, 9, and 10, and with the rigorous solution of Kaynia (1982). The main parameter is the shear-wave velocity ratio V_{s2}/V_{s1} of the two layers. Evidently, this ratio has an important effect, especially at high frequencies. Notice that a pile group in a homogeneous halfspace radiates more effectively than in a stratum with soil layers of

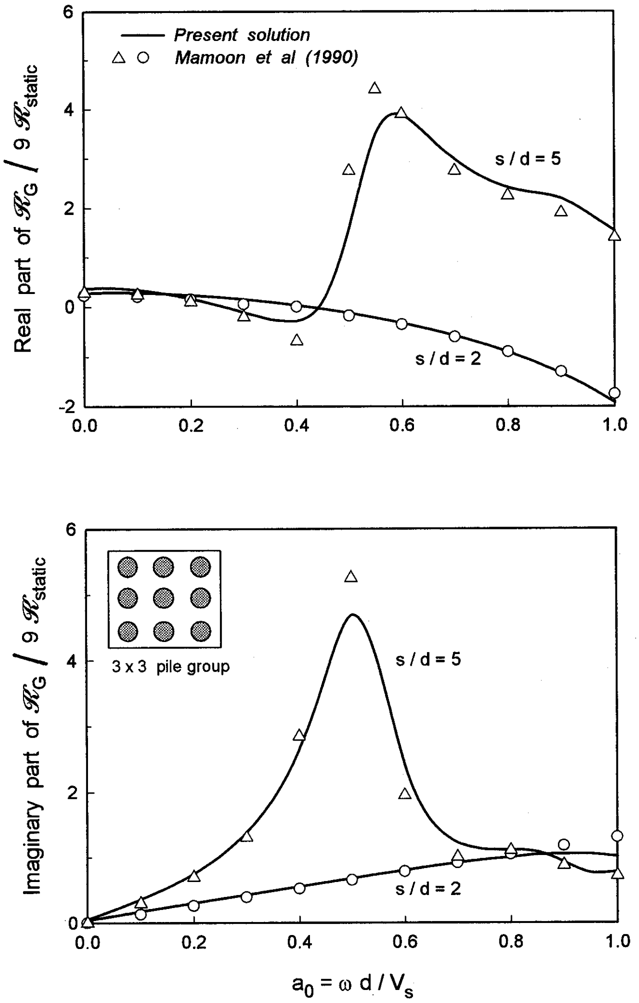

Fig. 13. Vertical dynamic stiffness and damping of a 3×3 pile group in a homogeneous halfspace: comparison of the proposed model with results form the rigorous solution of Mamoon et al.; $L/d=15$, $E_p/E_s=1000$, $\nu_s=0.40$, $\rho_p/\rho_s=1.40$, $\beta_s=5\%$

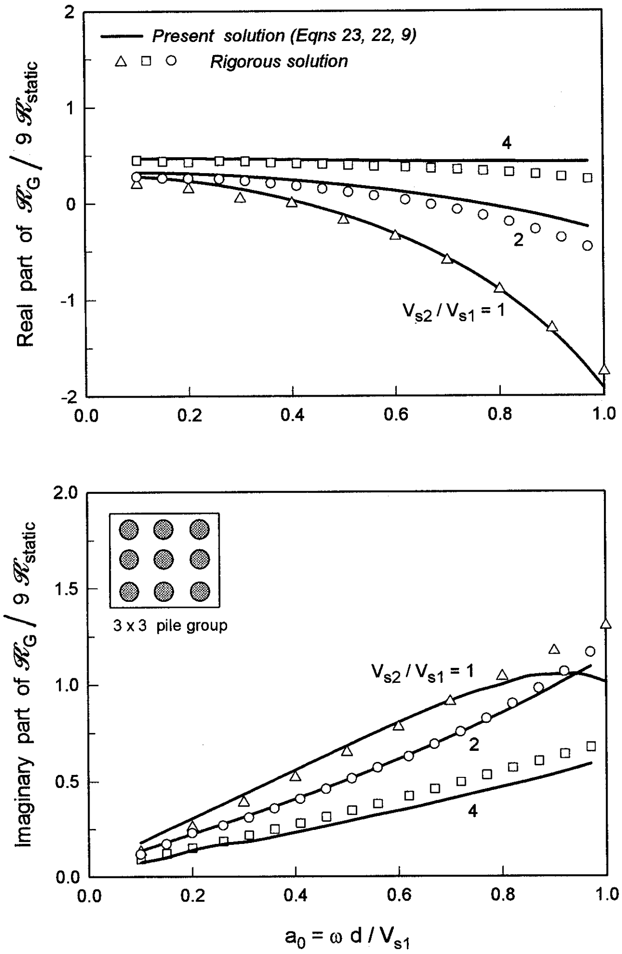

Fig. 14. Vertical dynamic stiffness and damping of a 3×3 pile group in two-layer soil for various shear-wave velocity ratios V_{s1}/V_{s2}: comparison of the present solution with results obtained using the rigorous method of Kaynia (1982); $s/d=2$, $L/d=20$, $h_1/L=2/3$, $E_p/E_{s1}=1000$, $\nu_s=0.40$, $\rho_p/\rho_{s2}=1.25$, $\rho_{s1}/\rho_{s2}=0.80$, $\beta_{s1}=10\%$, $\beta_{s2}=5\%$

very different stiffnesses. The accord with the rigorous method cannot be overstated.

ADDITIONAL AXIAL FORCE ALONG A PILE IN A GROUP

In the superposition method the actual group of m piles is conceptually replaced by two fictitious pile sets: (i) a set of m "source" piles carrying the actual pile loads P_i ($i=1, 2, \cdots m$), and (ii) a set of m "receiver" piles (carrying no load at their heads), but subjected to the waves generated by the "source" piles. Therefore, each actual pile in the group is conceptually represented by two substitute piles: the "source" pile and the "receiver" pile.

To account for pile-to-pile interaction, the superposition methods calculate the pile group impedance by adding the displacements atop the heads of each "source" and "receiver" pile pair. However, when calculating axial forces on piles, currently-used superposition solutions consider only the axial forces developed along the "source" piles. The axial forces developed along the "receiver" piles are overlooked. These additional forces (hereafter called "*additional pile distress*") are generated by the incoming waves in the form of dynamic shear stresses along the receiver piles. Despite the fact that in rigorous numerical formulations the additional pile distress is implicitly accounted for, its potential importance remains completely unexplored. The proposed model provides an simple way to calculate approximately the axial force profile along the grouped piles, including the "additional pile distress". For an arbitrary *pile i*, the procedure involves the following steps:

1. Determine the axial force profile along the "source" pile. Having determined the cap settlement D_G, the distribution of the cap load onto the individual piles is obtained through Eq. (23). The resulting load P_i atop pile i is the required boundary condition for calculating the force distribution, $P_i(z)$, along the "*source*" pile i. This can be done by using any pertinent analytical approach (such as the transfer-matrix formulation of Appendix II.)

2. Determine the displacement at the head of the *"receiver" pile*. If pile i were isolated, the vertical force P_i acting at the pile head would have produced a "source" displacement $W_i = P_i / \mathscr{R}$ atop the pile. However, the actual cap displacement, D_G, is in general different than W_i. The difference is the (positive or negative) additional displacement due to pile-to-pile interaction. Accordingly, an "additional" displacement δW_i can be defined which is equal to the difference between the actual cap displacement D_G and the "source" displacement W_i:

$$\delta W_i \equiv D_G - W_i \tag{24}$$

3. Determine the force and displacement profiles along the "receiver" pile. The additional displacement δW_i and the zero-force condition atop the receiver pile are the required boundary conditions for determining the additional force, $\delta P_i(z)$, and displacement, $\delta W_i(z)$, along that pile. To this end, Eq. (13) is rewritten in the following transfer-matrix form:

$$\begin{Bmatrix} \delta W_i(z) \\ \delta P_i(z) \end{Bmatrix} = [L_i(z)] \begin{Bmatrix} \delta W_i \\ 0 \end{Bmatrix} + \sum_{j=1}^{m, j \neq i} [Q(z)_{ij}] \begin{Bmatrix} W_j \\ P_j \end{Bmatrix} \tag{25}$$

where $[L]$ and $[Q]$ are 2×2 complex matrices given in Appendix II. The sum in the right-hand side of Eq. (25) corresponds to the forcing term induced by the "other" $(m-1)$ source piles. From this equation, the response along the "receiver" pile i can be readily calculated for any depth z.

4. Determine the total force and displacement profiles along the pile. The total axial force, $P_{\text{tot}}(z)_i$, along pile i is written as sum of the "source" force $P_i(z)$, plus the additional "received" force $\delta P_i(z)$:

$$P_{\text{tot}}(z)_i = P_i(z) + \delta P_i(z) \tag{26}$$

As an example, in the case of a simple *two-pile* group in homogeneous halfspace the additional force δP along each pile is obtained in closed form:

$$\delta P(z) = \frac{P_G \psi(s)}{4} \left\{ \frac{k_z + i\omega c_z}{k_z - m\omega^2 + i\omega c_z} \frac{E_p A_p \lambda}{\mathscr{R}} [\sinh(\lambda z) + \lambda z \cosh(\lambda z)] - \frac{k_z + i\omega c_z}{k_z - m\omega^2 + i\omega c_z} \lambda z \sinh(\lambda z) - 2 \frac{E_p A_p \lambda}{\mathscr{R}} \zeta(h\lambda) \sinh(\lambda z) \right\} \tag{27}$$

in which the single-pile stiffness, $\mathscr{R}$, is obtained from Eq. (1) and the diffraction function, $\zeta(h\lambda)$, from Eq. (15b).

Figure 15 plots the distribution with depth of axial pile force in an *edge* pile of a 3×3 group. The curves correspond to the amplitudes of: (i) the axial force $P(z)$ due to only the load *atop* the pile; (ii) the *additional axial force* $\delta P(z)$ due to pile-to-pile interaction; and (iii) the *total* axial force $P_{\text{tot}}(z)$. A general conclusion is that the additional axial force, $\delta P(z)$, is only a fraction of the force $P(z)$ from the load atop the pile. Notice, however, that at relatively low frequencies ($a_0 = 0.10$), which may be of particular interest in earthquake engineering, $\delta P(z)$ near the base of the pile may exceed $P(z)$. This implies that because of pile-to-pile interaction the load transmitted through the pile tip increases by a factor of 2 or more.

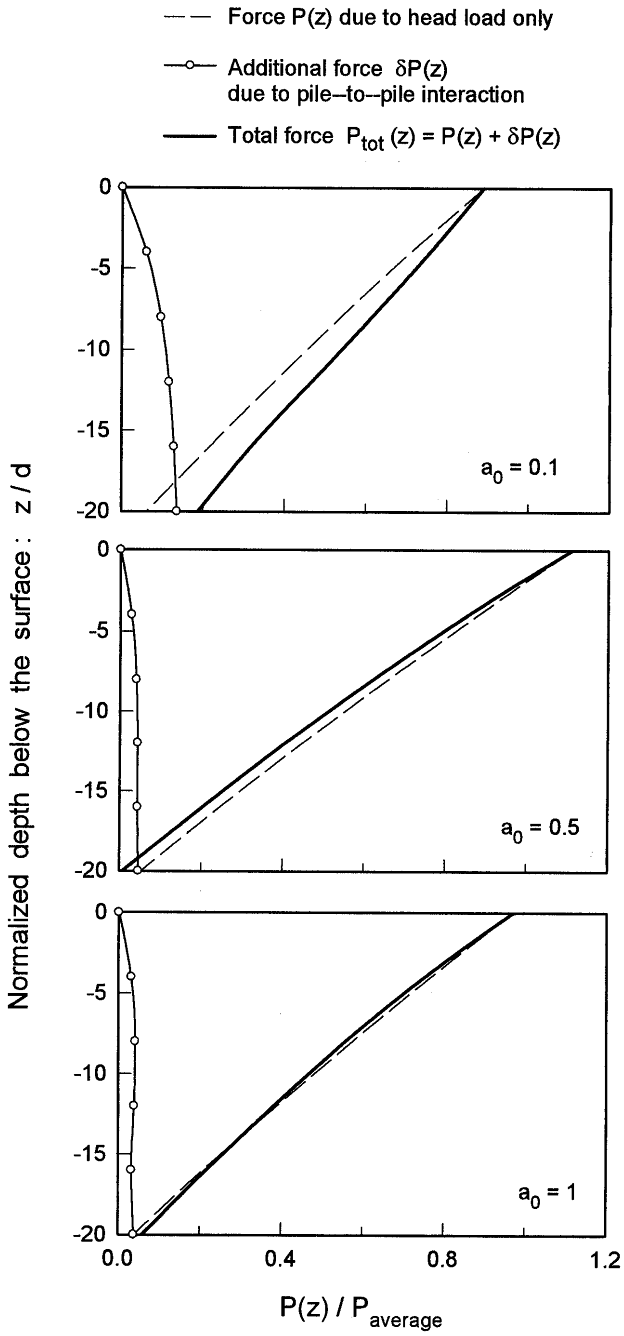

Fig. 15. Amplitude of the dynamic distribution of the axial force along an edge pile in a 3×3 group in homogeneous halfspace, for three different frequencies; $E_p/E_s = 1000$, $L/d = 20$, $\nu_s = 0.40$, $s/d = 5$

It is also worthy of note that at higher frequencies $P(z)$ and $\delta P(z)$ may be *out of phase* (*see* the case of $a_0 = 0.50$ in Fig. 15)-apparently, the result of *negative* interaction factors prevailing in the group. (This happens when the "average" distance between piles is about one-half of the wavelength.) Additional discussion can be found in Dobry and Gazetas (1988) and in the dissertation of Mylonakis (1995).

CONCLUSIONS AND LIMITATIONS

A simple physical method is presented for the dynamic response and internal forces of single piles and pile groups in homogeneous and layered soil. The method is based on a generalized dynamic Winkler model in conjunction with a "wave interference" solution for pile-to-pile interaction. The method permits the interaction factors for two-layered soils to be obtained in closed-form, and valuable insight to be gained in the physics of the problem. Dynamic interaction factors and group stiffnesses calculated with the proposed method are in convincing agreement with more rigorous solutions.

Limitations of the model stem from the simplifying assumptions of soil linearity (which leads to an upper-bound of the possible pile-to-pile interaction effect), and perfect bonding at the pile-soil interface. Moreover the superposition principle for pile groups is assumed valid for all pile groups-an assumption which may not be of sufficient accuracy when dealing with large closely-spaced pile groups, or when the piles are socketed in a very stiff bearing stratum, or when strongly non-linear soil effects dominate.

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APPENDIX I: DERIVATION OF DYNAMIC IMPEDANCE AT THE HEAD OF A SINGLE PILE IN HOMOGENEOUS SOIL STRATUM

Modeling the problem as an axially loaded beam supported on Winkler springs leads to the following differential equation:

$$E_pA_p\frac{d^2W(z)}{dz^2}+m\omega^2W(z)-(k_z+i\omega c_z)W(z)=0 \quad \text{(I-1)}$$

where $W(z)$ denotes the axial displacement of the pile. The general solution of Eq. (I-1) is

$$W(z)=Ae^{\lambda z}+Be^{-\lambda z} \quad \text{(I-2)}$$

where λ is given by Eq. (2) in the text. To eliminate the integration constants A and B, we enforce the boundary conditions at the top and bottom of the pile

$$z=0:\quad W(0)=1\exp[i\omega t] \quad \text{(I-3)}$$

$$z=L:\quad K_bW(L)=-E_pA_p\left(\frac{du}{dt}\right)_{z=L} \quad \text{(I-4)}$$

After straigthforward algebraic calculations we obtain:

$$\mathcal{R}=\frac{P(0)}{W(0)}=\frac{E_pA_p\lambda[K_b\cosh(h\lambda)+E_pA_p\lambda\sinh(h\lambda)]}{E_pA_p\lambda\cosh(h\lambda)+K_b\sinh(h\lambda)} \quad \text{(I-5)}$$

which reduces to the expression of Eq. (1) using the transformation of Ω given in Eq. (2). More details can be found in Mylonakis (1995).

APPENDIX II: TRANSFER MATRIX FORMULATION

For n soil layers, repeating the solution of the equation of motion, $W_{11}(z)=A_{11}\exp[\lambda z]+B_{11}\exp[-\lambda z]$ (*see* Appendix I), for each homogeneous layer while imposing the continuity of forces and displacements at each interface, we take:

$$\begin{Bmatrix} W_{11}(h) \\ P_{11}(h) \end{Bmatrix}_{bb}=[L]_b[L]_n[L]_{n-1}\cdots[L]_2[L]_1\begin{Bmatrix} W_{11}(0) \\ P_{11}(0) \end{Bmatrix}_1 \quad \text{(II-1)}$$

where the transfer matrices $[L]_b$ and $[L]_i$ are given by:

$$[L]_i=\begin{bmatrix} \cosh(\lambda_ih_i) & -(E_pA_p\lambda_i)^{-1}\sinh(\lambda_ih_i) \\ E_pA_p\lambda_i\sinh(\lambda_ih_i) & \cosh(\lambda_ih_i) \end{bmatrix} \quad \text{(II-2)}$$

$$[L]_b=\begin{bmatrix} 1 & -K_b^{-1} \\ 0 & 1 \end{bmatrix} \quad \text{(II-3)}$$

Enforcing the boundary conditions $W_{11}(h)_{bb}=0$ and $W_{11}(0)_1=1$, the stiffness of a solitary pile is easily obtained from (II-1) as $\mathcal{R}=P_{11}(0)_1$. Moreover, for a given force atop the pile, pile response at the bottom of any layer j can be calculated as:

$$\begin{Bmatrix} W_{11}(h) \\ P_{11}(h) \end{Bmatrix}_j=[L]_j[L]_{j-1}\cdots[L]_2[L]_1\begin{Bmatrix} -K^{-1} \\ 1 \end{Bmatrix}P_{11}(0) \quad \text{(II-4)}$$

In the case of two interacting piles, the transfer matrix equation connecting the top and bottom of an arbitrary segment of the "receiver" pile is (Mylonakis, 1995):

$$\begin{Bmatrix} W_{21}(h) \\ P_{21}(h) \end{Bmatrix}_i=[Q]_i\begin{Bmatrix} W_{11}(0) \\ P_{11}(0) \end{Bmatrix}_i+[L]_i\begin{Bmatrix} W_{21}(0) \\ P_{21}(0) \end{Bmatrix} \quad \text{(II-5)}$$

in which $[L]$ is given by Eq. (II-2) and $[Q]$ by

$$[Q]_i=(k_{z_i}+i\omega c_{z_i})\frac{\psi(s)}{2\lambda_i}\begin{Bmatrix} -\dfrac{h_i}{E_pA_p}\sinh(h_i\lambda_i) & \dfrac{1}{(E_pA_p)^2\lambda_i}\left[h_i\cosh(h_i\lambda_i)-\dfrac{\sinh(h_i\lambda_i)}{\lambda_i}\right] \\ h_i\lambda_i\cosh(h_i\lambda_i)+\sinh(h_i\lambda_i) & -\dfrac{h_i}{E_pA_p}\sinh(h_i\lambda_i) \end{Bmatrix} \quad \text{(II-6)}$$